Optimization of Unconstrained Problems with *N*-Variables

• Random Search

- One of the most inefficient but easiest to implement search methods for *N*-dimensional optimization problems,

$$x_i = x_i^l + r(x_i^u - x_i^l)$$

  - where x_i^l and x_i^u are the lower and upper bounds on the design variables, respectively, and r is a real random number between 0 and 1.0.

• One-Dimensional Minimization

- Most n-dimensional search algorithms use one dimensional minimization to determine the minimum along a specified direction,

$$\boldsymbol{x} = \boldsymbol{x}^q + \alpha \boldsymbol{d}^q$$

  - where $\boldsymbol{x}^q$ is the current point $\boldsymbol{d}^q$ is the direction vector at that point, and α is the variable which determines how far one needs to move along $\boldsymbol{d}^q$ to reach minimum.

- The function to be minimized, *f(x)*, can now be expressed in terms of the variable α.

Minimize $f(\boldsymbol{x}) = f(\boldsymbol{x}^q + \alpha \boldsymbol{d}^q) = f(\alpha)$

at minimum $\alpha = \alpha^*$

- Methods for 1-D Minimization
  - Bracketing
  - Polynomial Approximations
  - Golden Section Search

• Bracketing (Fig 2-11, Vanderplaats)

- We assume that the solution lies in the positive domain, and the function has a minimum.
- Initialize $\alpha_L = 0.0$, small $\Delta\alpha$, $a = (1+\sqrt{5})/2$ (Gold. Sect. Ratio), and α_{max}.
- Compute lower and upper bound values of the interval and the function at those points: $f_L = f(\alpha_L)$, $\alpha_U = \alpha_L + \Delta\alpha$, and $f_U = f(\alpha_U)$
- If $f_U > f_L$, minimum is bracketed, exit. Other wise,
- Let $\alpha_1 = \alpha_U$ and $f_1 = f_U$. Compute $\alpha_U = (1+a)\alpha_1 - a\,\alpha_L$
- If $\alpha_U > \alpha_{max}$, exit unbounded.
- If not, compute $f_U = f(\alpha_U)$.
- If $f_U > f_1$, minimum is bracketed, exit
- If not, let $\alpha_L = \alpha_1$ and $f_L = f_1$, and go to 5th bullet to repeat the process.

• Polynomial Approximations

- Evaluate the function to be minimized and possibly its derivatives at several points and then fit a polynomial of desired order to those points.
- Depending on the order of the polynomial, the minimum of the polynomial can easily be evaluated, and would produce a good estimate of the minimum of the actual function.
- Different possible approximations and the information needed
  - 2-Point Quadratic: f_1, f_2 and either f'_1 or f'_2
  - 3-Point Quadratic: f_1, f_2 and f_3
  - 3-Point Cubic: f_1, f_2, f_3, and derivative at one of the points
  - 4-Point Cubic: f_1, f_2, f_3, and f_4

• Golden Section Search

- Assume the minimum is bracketed between α_L and α_U, and the function is unimodal in the interval.
- Compute 2 intermediate points: $\alpha_1=\alpha_L + \tau\,(\alpha_U - \alpha_L)$, $\alpha_2=\alpha_U - \tau\,(\alpha_U - \alpha_L)$ where $\tau = 1- 1/a = (3-\sqrt{5})/2$ and a = Golden Section Ration
- Either α_1 or α_2 will be the new bound on the minimum
- If $f_1 > f_2$, then α_1 forms the new lower bound, replacing α_L
- If $f_2 > f_1$, then α_2 forms the new upper bound, replacing α_U
- The remaining point will be either α_1 or α_2 depending on the case above
- A new α_1 or α_2 and associated function value will be computed by using the equations above